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TRIPPING OF STIFFENED PLATES USING
A REFINED BEAM THEORY

by

Donald Danielson

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TRIPPING OF STIFFENED PLATES USING A REFINED BEAM THEORY

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ABSTRACT

The subject of this paper is the buckling behavior of thin bar stiffeners attached to plates which are subjected to normal pressure loadings. The stiffeners are mathematically modeled by a nonlinear beam theory recently derived. An analytical solution is obtained to the beam buckling equations. The curvature of the centerline along the beam's base at the buckling load is expressed in terms of the beam's thickness, height, and rotational stiffness. Analytical results are compared with an experiment recently performed.

I. INTRODUCTION: STATE OF THE ART

Stiffened plates and shells are a basic structural component of submarines, ships, and aircraft. These structures are designed with generous safety margins against overall collapse triggered by frame yielding or <u>tripping</u>. Tripping is a lateral torsional buckling occurring in flexurally stiff frames which have low lateral rigidity. The object of analytical work is to determine design criteria to inhibit tripping at any stress less than yield. Tripping reduces structural integrity and may initiate failure of the entire structure by general buckling.

Surprisingly little material exists in the literature on the subject of the lateral instability of stiffeners welded to continuous plating. There are few studies based on theories simple enough to have analytical solutions. Earlier work is summarized by Bleich (1952). Kennard (1959) studied initially curved stiffeners. Adamchak (1979,1982) pointed out the importance of rotational constraint on the buckling load. Van der Neut (1982) developed a theory for a Z-stiffened panel in compression that could be solved with a pocket calculator. More accurate codes requiring powerful computers were developed by Smith (1968) and Wittrick (1968) based on folded plate analysis. Bushnell (1985) also modeled the rings on cylindrical shells as plates.

In the past there have been few experiments in which stiffeners attached to shells have been allowed to buckle. Smith (1975) tested the compressive strength of ship grillages.

Recently, at the Naval Postgraduate School, a series of stiffened plates have been subjected to static or dynamic pressure loads sufficient to cause tripping. Each plate was rectangular in cross section and fixed at its boundaries. A single narrow-flanged stiffener was attached at its base to the center of each plate and free at its ends. Measurements were made of strains and deflections versus pressure, as reported by Budweg and Shin (1987). At low pressures a plate-stiffener simply bowed out symmetrically, but above a critical buckling pressure the stiffener rotated about its base and deformed unsymmetrically (see Fig. 1).

In this paper we mathematically simulate these experiments. Our analysis is based upon the following assumptions:

(i) The stiffener is rectangular in cross section with thickness t, height h, and length ℓ. $\frac{t^2}{h^2}$ and $\frac{h^2}{\ell^2}$ are negligible compared to 1.

(ii) The reference line along the center of the stiffener's base undergoes a vertical displacement w and negligible horizontal displacement. $\frac{w}{h}$ is negligible compared to 1.

(iii) Each stiffener cross section does not distort in its plane and remains normal to the reference line.

(iv) The deformation normal to the plane of a stiffener cross section is equal to the product of the warping function of the Saint-Venant torsion theory times the twist of the stiffener.

(v) The stiffener material is linear, isotropic and elastic.

(vi) The plate does not participate in the buckling of the stiffener.

(vii) At the buckling load the curvature of the reference line is a constant.

Assumptions (i)-(v) are specializations of the ones previously used in deriving a refined nonlinear beam theory used to model helicopter rotor blades (Danielson and Hodges, 1987, 1988). Hence we can use that theory for the present problem. Assumptions (vi)-(vii) uncouple the beam problem from the plate problem. Thus the mathematical model reduces to one dimension, in contrast to previous analyses in which the stiffener-plate was modeled by two-dimensional plate theories. As a consequence of the simpler formulation, we will be able to obtain an analytical solution to the equations.

II. NONLINEAR BEAM EQUATIONS

In this section we reproduce relevant formulas from our previous papers (Danielson & Hodges, 1987,1988), applied to the present problem. We retain the same notation as in these earlier papers.

The centerline along the base of the undeformed beam is called the reference line r (refer to Fig. 2). The Cartesian coordinates of a point in the undeformed beam are denoted by (x_1, x_2, x_3), where x_1 denotes distance measured along r from the middle of the base, x_2 denotes distance measured normal to r parallel to the plate, and x_3 denotes distance measured normal to the plate from the base. At each point an orthogonal reference triad $(\mathbf{b}_1^r, \mathbf{b}_2^r, \mathbf{b}_3^r)$ tangent to the coordinate curves is defined, with $\mathbf{b}_1^r$ parallel to the x_1 axis. The position vector to points in the undeformed beam is then given by

$$\mathbf{r} = x_i \mathbf{b}_i^r \tag{1}$$

(The repeated index i is summed from 1 to 3.)

After deformation the locus of material points on the reference axis is denoted by R. Now at each point along R define an orthogonal reference triad $(\mathbf{b}_1^R, \mathbf{b}_2^R, \mathbf{b}_3^R)$ tangent to the <u>deformed</u> coordinate curves. Also, let $\mathbf{b}$ denote an intermediate unit vector in the direction of the principal normal to R. It follows from assumptions (i)-(iii) that

$$\mathbf{b}_1^R = \mathbf{b}_1^r + w'\mathbf{b}_3^r$$

$$\mathbf{b} = \mathbf{b}_3^r - w'\mathbf{b}_1^r$$

$$\mathbf{b}_2^R = \cos\theta \ \mathbf{b}_2^r + \sin\theta \ \mathbf{b} \qquad (2)$$

$$\mathbf{b}_3^R = -\sin\theta \ \mathbf{b}_2^r + \cos\theta \ \mathbf{b}$$

Here θ is the angle of rotation between $\mathbf{b}$ and $\mathbf{b}_3^R$, and primes denote differentiation with respect to x_1. Expanding $\cos\theta$ and $\sin\theta$ in powers of θ and retaining up to quadratic terms (higher order terms are not needed for the buckling analysis), we obtain from (2)

$$\mathbf{b}_2^R = -w'\theta\,\mathbf{b}_1^r + (1-\frac{\theta^2}{2})\mathbf{b}_2^r + \theta\mathbf{b}_3^r$$

$$\qquad (3)$$

$$\mathbf{b}_3^R = -(1-\frac{\theta^2}{2})w'\mathbf{b}_1^r - \theta\,\mathbf{b}_2^r + (1-\frac{\theta^2}{2})\mathbf{b}_3^r$$

The curvature vector of the deformed beam is defined by

$$\mathbf{K} = \kappa_i\mathbf{b}_i^R = \frac{1}{2}\mathbf{b}_i^R \times (\mathbf{b}_i^R)' \qquad (4)$$

The components of the curvature vector are obtained from (2)-(4):

$$\kappa_1 = \theta'$$

$$\kappa_2 = -w'' + \frac{w''\theta^2}{2} \qquad (5)$$

$$\kappa_3 = w''\theta$$

It follows from assumptions (i)-(iv) that the position vector to points in the deformed beam is given by

$$\mathbf{R} = x_1\mathbf{b}_1^r + w\mathbf{b}_3^r + x_2\mathbf{b}_2^R + x_3\mathbf{b}_3^R + x_2x_3\theta'\mathbf{b}_1^R \tag{6}$$

Simplified expressions for the extensional strain γ_{11} and transverse shear strains γ_{12} and γ_{13} are derived in our earlier papers. Below we reproduce equations (9)-(10) from Danielson and Hodges (1988), specialized to the present problem:

$$\begin{aligned}
\gamma_{11} &= E_{11} + E_{12}\phi_3 - E_{13}\phi_2 + \frac{1}{2}\phi_2^2 + \frac{1}{2}\phi_3^2 \\
\gamma_{12} &= E_{12} - \frac{1}{2}E_{11}\phi_3 \\
\gamma_{13} &= E_{13} + \frac{1}{2}E_{11}\phi_2
\end{aligned} \tag{7}$$

$$\begin{aligned}
E_{11} &= x_3\kappa_2 - x_2\kappa_3 + x_2x_3\theta'' \\
E_{12} &= \frac{-x_3\kappa_1 + x_3\theta' + x_2x_3\theta'\kappa_3}{2} \\
E_{13} &= \frac{x_2\kappa_1 + x_2\theta' - x_2x_3\theta'\kappa_2}{2} \\
\phi_2 &= \frac{-x_2\kappa_1 + x_2\theta' + x_2x_3\theta'\kappa_2}{2} \\
\phi_3 &= \frac{-x_3\kappa_1 - x_3\theta' + x_2x_3\theta'\kappa_3}{2}
\end{aligned} \tag{8}$$

Substituting (5) and (8) into (7), and neglecting small and higher order terms, we obtain

$$\gamma_{11} = -x_3 w'' + x_2 x_3 \theta'' + \frac{x_3^2(\theta')^2}{2} - x_2 w''\theta + \frac{x_3 w''\theta^2}{2} + \frac{x_2^2 x_3 w''(\theta')^2}{2}$$

$$\gamma_{12} = \frac{-x_3^2 w''\theta'}{2} + \frac{x_2 x_3^2 \theta'\theta''}{2} \qquad (9)$$

$$\gamma_{13} = x_2\theta' + \frac{x_2 x_3 w''\theta'}{2} - \frac{x_2^2 x_3^2 w''\theta'\theta''}{4}$$

The other strain components ($\gamma_{22}, \gamma_{23}, \gamma_{33}$) turn out to be negligible in the strain energy expression.

It follows from assumption (v) that the strain energy of the stiffener is

$$W_s = \int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} \int_{-\frac{t}{2}}^{\frac{t}{2}} \int_0^h \frac{E}{2}\left(\gamma_{11}^2 + \frac{2\gamma_{12}^2}{1+\nu} + \frac{2\gamma_{13}^2}{1+\nu}\right) dx_3\, dx_2\, dx_1 \qquad (10)$$

where E is Young's modulus and ν is Poisson's ratio. Substituting (9) into (10), performing the x_2 and x_3 integrations, and neglecting small and higher order terms, we obtain

$$W_s = \int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} \frac{Eh^3 t(w'')^2}{6}\, dx_1 + Q \qquad (11)$$

Here Q denotes the terms in W_s which are quadratic in θ and its derivatives:

$$Q = \int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} \frac{E}{4}\left[\frac{t^3h^3}{18}(\theta'')^2 - \frac{th^4w''}{2}(\theta')^2 + \frac{t^3h}{3(1+\nu)}(\theta')^2\right] dx_1 \tag{12}$$

III. BUCKLING EQUATIONS

We base our buckling analysis on the energy criterion of elastic stability. This criterion and its application are explained by Timoshenko and Gere (1961), Danielson (1974) and Wempner (1981).

The total potential energy of the plate-stiffener combination is the functional

$$P[u, \theta] = W_p[u] + W_s[w'', \theta] - V[u] \tag{13}$$

Here u denotes the displacement of points on the top surface of the plate; note that the vertical displacement of the plate along the reference line is equal to w. W_p denotes the strain energy of the plate; W_p is a functional of u. W_s is the strain energy of the beam, given by (11)-(12); W_s is a functional of the beam curvature w" and rotation θ. V is the work of the external loads, which is the hydrostatic pressure times the volume between the undeformed and deformed plate; V is a functional of the plate displacement field u only.

The <u>prebuckling</u> equilibrium state I in the plate-stiffener is denoted by $(u, w'', \theta = 0)$. The potential energy in the prebuckling state I is thus

$$P_I = W_p[u] + W_s[w'', 0] - V[u] \tag{14}$$

Invoking assumption (vi), we consider a small deviation $\theta \neq 0$ from the prebuckling state of the plate-stiffener. The potential energy in this alternate state II is

$$P_{II} = W_p[u] + W_s[w'', \theta] - V[u] \tag{15}$$

From (11), (14) and (15), the <u>change</u> in potential energy is thus

$$P_{II} - P_I = Q[w'', \theta] \tag{16}$$

According to the energy criterion of elastic stability, when the curvature reaches a critical value $\bar{w}''$, there exists a bifurcation buckling mode $\bar{\theta}$ satisfying

$$Q[\bar{w}'', \bar{\theta}] = 0, \qquad Q[\bar{w}'', \theta \neq \bar{\theta}] > 0 \tag{17}$$

As a consequence of (17), the buckling mode is determined by the variational equation

$$\delta Q = 0 \tag{18}$$

Taking the variation of (12), and integrating by parts, we obtain

$$\delta Q = \left[\frac{Et^3h^3}{36} \bar{\theta}''\delta\theta'\right]_{-\frac{\ell}{2}}^{\frac{\ell}{2}} - \int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} \frac{E}{2}\left[\frac{t^3h^3}{18} \bar{\theta}''' + \frac{th^4\bar{w}''}{2} \bar{\theta}' - \frac{t^3h}{3(1+\nu)}\bar{\theta}'\right]\delta\theta' dx_1 = 0 \tag{19}$$

In order for (19) to vanish for arbitrary $\delta\theta'$, $\bar{\theta}$ must satisfy the differential equation

$$\bar{\theta}''' + \left(\frac{9h\bar{w}''}{t^2} - \frac{6}{(1+\nu)h^2}\right)\bar{\theta}' = 0 \tag{20}$$

and the boundary conditions

$$\bar{\theta}''\left(\frac{\ell}{2}\right) = \bar{\theta}''\left(-\frac{\ell}{2}\right) = 0 \tag{21}$$

In order to obtain an analytical solution to the differential equation (20), we invoke assumption (vii). The solution to the eigenvalue problem (20)-(21) is then

$$\bar{\theta}' = \begin{cases} A\cos\left[\dfrac{2n\pi x_1}{\ell}\right] \\ \text{or} \\ B\sin\left[\dfrac{(2n+1)\pi x_1}{\ell}\right] \end{cases} \tag{22}$$

where A and B are arbitrary constants and n is any positive integer. Furthermore, the curvature is

$$\bar{w}'' = \frac{2t^2}{3(1+\nu)h^3} + \underline{\frac{n^2\pi^2t^2}{9h\ell^2}} \tag{23}$$

For small values of n, the underlined term in (23) is negligible. Hence the wavelength of the buckling mode is not uniquely determined. The critical curvature of the reference line at the buckling load is thus

$$\bar{w}'' = \frac{2t^2}{3(1+\nu)h^3} \tag{24}$$

The maximum value of the compressive strain at the buckling load is obtained by setting $\theta = 0$ and $x_3 = h$ in the first of equations (9):

$$|\gamma_{11}|_{max} = h\bar{w}'' \tag{25}$$

IV. ROTATIONAL CONSTRAINT

In the preceding analysis the beam cross section was allowed to rotate freely about the reference axis (simple support). In this section we determine the effect of rotational constraint on the buckling load. The method used is explained by Timoshenko and Woinowsky-Krieger (1959) and Ugural (1981).

Resistance to rotation can be modeled by considering the base of the beam to be supported by a foundation, itself assumed to experience elastic deformation. We see from (3) and (6) that a point on the base of the beam undergoes a vertical deflection $(w + x_2\theta)$. The foundation reaction forces are assumed to be $K(w + x_2\theta)$. Here K is a constant called the modulus of the foundation and has the dimensions of force per unit area of the base per unit deflection. The strain energy W_f due to deformation of the foundation is then

$$W_f[w, \theta] = \frac{K}{2} \int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} \int_{-\frac{t}{2}}^{\frac{t}{2}} (w + x_2\theta)^2 \, dx_2 \, dx_1$$

$$= \frac{Kt}{2} \int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} w^2 \, dx_1 + \underline{\frac{Kt^3}{24} \int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} \theta^2 \, dx_1} \qquad (26)$$

It follows from the arguments in the preceding sections that the underlined term in (26) must be added to the expression (12) for the quadratic terms in the change of potential energy Q. Then solving (17) for the critical curvature, we obtain

$$\bar{w}'' \leq \frac{\int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} \{\frac{E}{4}[\frac{t^3h^3}{18}(\theta'')^2 + \frac{t^3h}{3(1+\nu)}(\theta')^2] + \frac{Kt^3}{24}\theta^2\} dx_1}{\frac{Eth^4}{8}\int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} (\theta')^2 \, dx_1} \tag{27}$$

We assume that the buckling mode can be approximated by our previous solution (22), which is exact when $K = 0$:

$$\theta = A \sin[\frac{2n\pi x_1}{\ell}] \tag{28}$$

where A is an arbitrary constant and n is any positive integer. Substitution of (28) into (27) then yields bounds on the critical curvature:

$$\frac{2t^2}{3(1+\nu)h^3} \leq \bar{w}'' \leq \frac{2t^2}{3(1+\nu)h^3} + \underline{\frac{4t^2\pi^2}{9h\ell^2}(n^2 + \frac{C^2}{n^2})} \tag{29}$$

where

$$C = \frac{1}{4\pi^2}\sqrt{\frac{3K\ell^4}{Eh^3}} \tag{30}$$

The best estimate for $\bar{w}''$ is obtained by choosing the value of n which minimizes the underlined term in (29). When $C < 2$, the minimum value occurs for $n = 1$, and the underlined term is negligible. When $C > 2$, the minimum value occurs for $n > 1$, but is still negligible until n^2 is large. When n^2 is large, we can treat n^2 as a continuous variable and set the derivative with respect to n^2 of the underlined term equal to zero, which results in a value of $n^2 = C$. Substitution of $n^2 = C$ and (30) into (29) then yields the best bounds

$$\boxed{\frac{2t^2}{3(1+\nu)h^3} \leq \bar{w}'' \leq \frac{2t^2}{3(1+\nu)h^3} + \frac{2t^2}{9h^3}\sqrt{\frac{3Kh}{E}}} \tag{31}$$

V. EXPERIMENT

In this section we compare our formulas with an experiment performed at the Naval Postgraduate School. The experimental results are taken from Budweg (1986) and Budweg and Shin (1987).

A rectangular plate with a narrow-flanged stiffener was machined out of a single large blank of 6061-T6 aluminum. The dimensions of the resulting plate and stiffener are shown in Fig. 3. A strongback was bolted to the test panel. The test panel cavity was gradually filled with water. Measurements of the strains and deflections at the bottom of the plate, and the strain at the top of the stiffener, were taken at various hydrostatic pressures.

The experimenters judged that tripping of the stiffener initiated at a deflection of about three plate thicknesses, when the curvature of the plate at the stiffener location and the compressive strain at the top of the beam was approximately .013. The plate was loaded to a maximum vertical deflection of about four plate thicknesses. It was observed that the vertical deflection of the plate was always symmetric about the plate's center lines. After release of the pressure there was permanent plastic deformation remaining in the plate, but no deformation of the stiffener out of the vertical plane.

Let us now calculate the critical curvature obtained from our analysis which ignores rotational constraint. Substituting $\nu = .33$ and the dimensions of the stiffener cross section shown in Fig. 3 into the formulas (24)-(25), we predict

$$\bar{w}'' = \frac{2(.125)^2}{3(1.33)(1.125)^3} = .0055 \quad \text{(units of inch}^{-1}\text{)}$$

$$|\gamma_{11}|_{max} = (1.125)(.0055) = .0062 \tag{32}$$

The predicted critical curvature and strain are less than half of the measured values .013.

Let us examine the factors we have neglected in our analysis, to see which could create a significant increase in the predicted buckling load:

(i) The beam cross section was not rectangular and the neutral axis in bending was not at the top of the plate. However, fitting the cross section with a more accurate T-shape, and assuming the beam is 15/16" high (neglecting the fillet at the base of the beam), we obtain a critical curvature of only .0061.

(ii) The prebuckling state of the stiffener was nonlinear so larger vertical and nonnegligible horizontal displacements occur. But replacing assumption (ii) by the less stringent assumption that $w/h < 1$, and repeating the derivation in section III retaining nonlinear terms in w, we find no substantial change in (32).

(iii) The critical curvature $\bar{w}''$ was not constant. However, the experimental measurements showed significant variation in $\bar{w}''$ only near the ends of the stiffener.

(iv) The structure had geometrical and material imperfections. But imperfections usually lower the buckling load.

(v) The beam cross section deforms in its plane. But inplane extension and contraction have negligible effects on the critical curvature. And allowing the cross section to bend cannot increase the buckling load.

(vi) The stiffener was partially restrained by the plate against rotation at its base. Remembering the result (31) of section IV, we must conclude that the rotational restraint is large enough to be a significant factor in the tripping of the tested plate-stiffener.

We can estimate the magnitude of the rotational restraint by solving (31) for K:

$$K \geq \frac{E}{3h}\left(\frac{9h^3\bar{w}''}{2t^2} - \frac{3}{1+\nu}\right)^2 \qquad (33)$$

Substituting $\bar{w}'' = .013$ and the beam dimensions into (33), we obtain

$$K \geq 3E \qquad (34)$$

VI. CONCLUSION

As a consequence of the theoretical formula (31), we can draw the following conclusions about the tripping behavior of stiffened plates:

1. The critical curvature $\overline{w}''$ does not depend on the length ℓ of the beam.
2. Beams with smaller thickness to height ratios t/h trip at smaller curvatures.
3. The tripping point depends very much on the restraining stiffness K.

Our mathematical analysis has treated only the stiffener and has not considered its interaction with the supporting structure. The analysis could be improved by solving the prebuckling problem for the plate-stiffener, a task which is best done on a computer. Additional kinematical and material nonlinearities could be included. The pressure which causes tripping could be calculated and compared with experiment.

Further experiments need to be done. It was difficult to determine when and if tripping occurred from the measurements that were made. Future experiments should allow visual inspection of a stiffener while it is buckled. Additional experimental measurements could be made to determine the rotational stiffness.

The advantage of the present methods is that they lead to simple analytical formulas which reveal the dependence of the buckling point on geometrical parameters. It would be interesting to see if these analytical techniques could be applied to other problems, such as the buckling of initially curved shells with stiffeners of T or Z cross sections.

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REFERENCES

Adamchak, J.C., 1979, "Design Equations for Tripping of Stiffeners Under Inplane and Lateral Loads," David W. Taylor Naval Ship Research and Development Center Report 79/064.

________, 1982, "ULSTR: A Program for Estimating the Collapse Moment of a Ship's Hull Under Longitudinal Bending," David W. Taylor Naval Ship Research and Development Center Report 82/076.

Bleich, F., 1952, Buckling Strength of Metal Structures, McGraw-Hill.

Budweg, H.L., 1986, An Investigation into the Tripping Behavior of Longitudinally T-Stiffened Rectangular Flat Plates Loaded Statically and Impulsively, Master's Thesis, Naval Postgraduate School, Monterey, California.

Budweg, H.L. and Shin, Y.S., 1987, "Experimental Studies on the Tripping Behavior of Narrow T-Stiffened Flat Plates Subjected to Hydrostatic Pressure and Underwater Shock," The Shock and Vibration Bulletin, to appear.

Bushnell, D., 1985, Computerized Buckling Analysis of Shells, Martinus Nijhoff.

Danielson, D.A., 1974, "Theory of Shell Stability," in Thin-Shell Structures: Theory, Experiment and Design, edited by Y.C. Fung and E.E. Sechler, Prentice-Hall.

Danielson, D.A., and Hodges, D.H., 1987, "Nonlinear Beam Kinematics by Decomposition of the Rotation Tensor," ASME Journal of Applied Mechanics, Vol. 54, pp. 258-262.

________________, 1988, "A Beam Theory for Large Global Rotation, Moderate Local Rotation, and Small Strain," ASME Journal of Applied Mechanics, to appear.

Kennard, E.H., 1959, "Tripping of T-Shaped Stiffening Rings on Cylinders Under External Pressure," David Taylor Model Basin Report 1079.

Smith, C.S., 1968, "Bending, Buckling and Vibrations of Orthotropic Plate-Beam Structures," Journal of Ship Research, Vol. 12, No. 5, pp. 249-268.

________, 1975, "Compressive Strength of Welded Ship Grillages," Naval Construction Research Establishment (Great Britain) R611.

Timoshenko, S.P., and Woinowsky-Krieger, S., 1959, Theory of Plates and Shells, McGraw-Hill.

Timoshenko, S.P., and Gere, J.M., 1961, Theory of Elastic Stability, McGraw-Hill.

Ugural, A.C., 1981, Stresses in Plates and Shells, McGraw-Hill.

Van der Neut, A., 1982, "Overall Buckling of Z-Stiffened Panels in Compression," in Collapse: The Buckling of Structures in Theory & Practice, edited by J.M.T. Thompson and G.W. Hunt, Cambridge University Press.

Wempner, G.A., 1981, Mechanics of Solids with Application to Thin Bodies, Sijthoff & Noordhoff, the Netherlands.

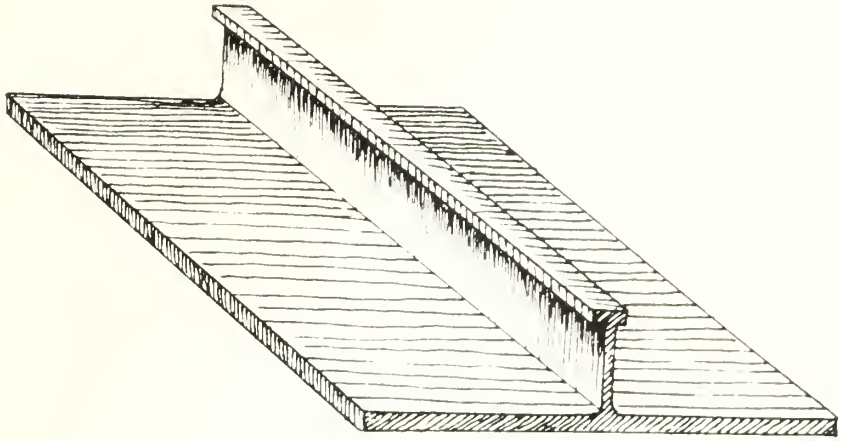
Wittrick, W.H., 1968, "General Sinusoidal Stiffness Matrices for Buckling & Vibration Analyses of Thin Flat-Walled Structures," International Journal of Mechanical Sciences, Vol. 10, No. 12, pp. 949-966.

FIGURE CAPTIONS

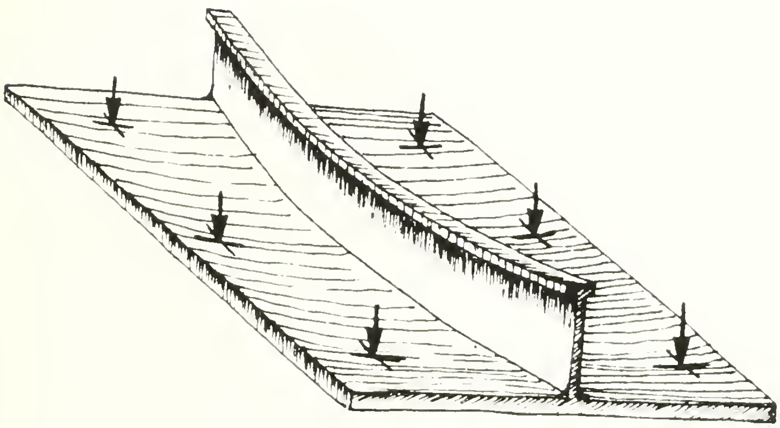
Figure 1: Stiffener shapes when plate is under various pressures

Figure 2: Geometry used in mathematical analysis

Figure 3: Dimensions of experimental model



1. UNDEFORMED STATE



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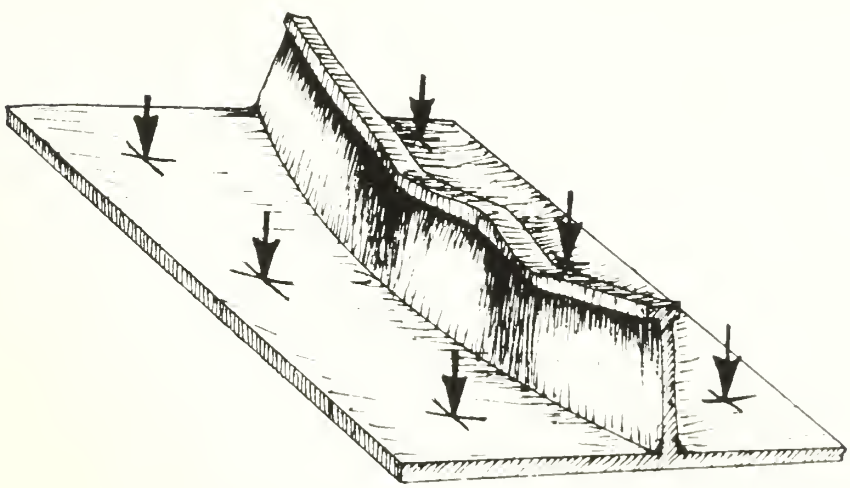

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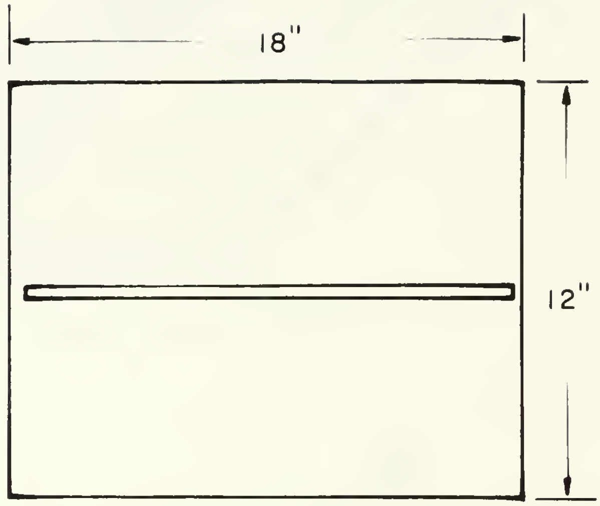
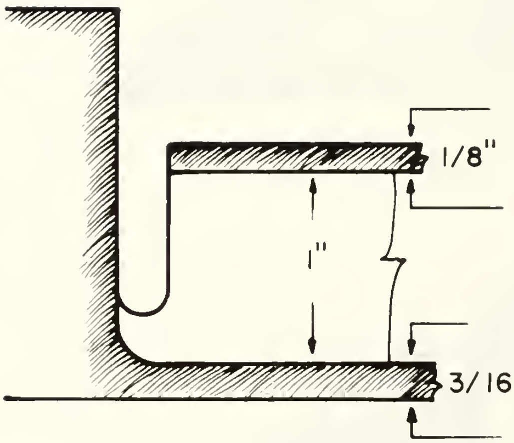
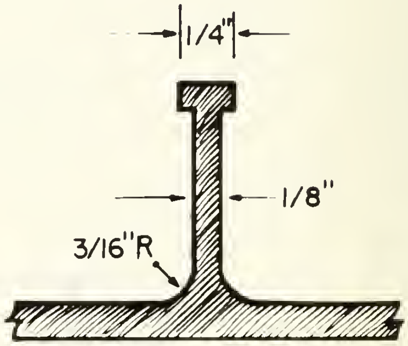


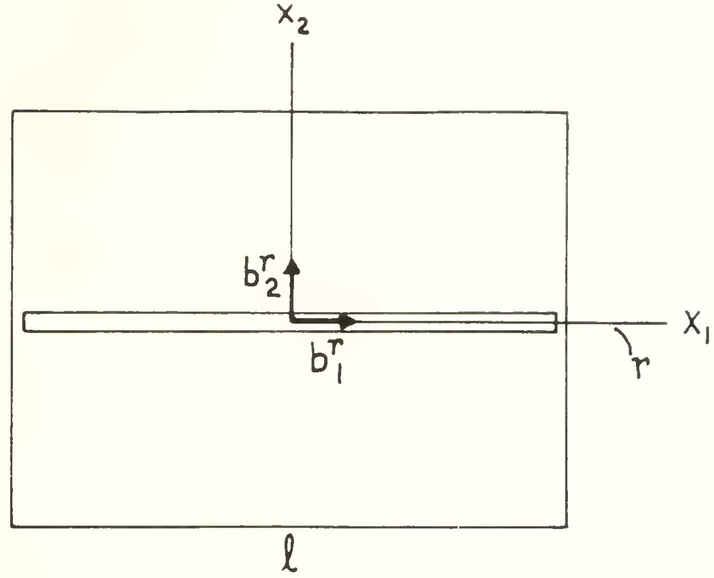
TOP VIEW



SIDE VIEW



END VIEW



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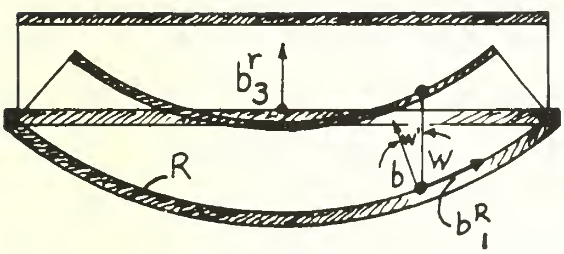
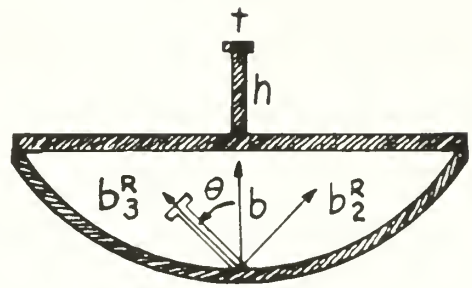


SIDE VIEW



END VIEW

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